# Temporal Discretization – Introduction

-Introduction paragraph  
 -Euler method (First order) (Piecewise Continuous)  
 -Explain Numerical Diffusion and Drift

The Midpoint Aproximation and Runge-Kutta methods are examples of higher order schemes, however they both multiple recalculations of blob velocities for a single time step, whilst increasing the accuracy of the simulation this is computationally expensive.

# Temporal Discretization - First Order Downwind Scheme – Theory

The Biot-Savart law results in the calculation of the velocities of all blobs, the first order scheme makes the assumption that this velocity acts for the entire time period until the next iteration of the Biot-Savart law. So it assumes the downwind velocity of the fluid at the next time step is the current fluid velocity. This is shown in equation (1) where vxt is the x velocity at the current time step and

Solving equation (1) leads to equation (2)

If the time is taken to be 0 at the current time step, this scheme simplifies to equation (3) where vx is the current x-velocity.

This scheme is applied to all 3 spatial dimensions to obtain a new position vector for every blob, this is shown in vector form in equation (4)

# Temporal Discretisation –Quadratic Positional Scheme – Theory

This scheme approximates the position at the next time step by use of a polynomial to estimate the position of an element at t+ts. To map a second degree polynomial the current position is used, the position at the previous time step and the current velocity. This leads of equations (1) (2) and (3) respectively.

Rearranging (1) and (2) for B and equation the two yields equation (4)

Equation (3) can be rearranged for B, this is shown in equation (5)

Substituting equation (5) into equation (4) yields equation (6)

Collecting terms of A:

# Temporal Discretization – Quadratic Interpolation Scheme - Theory

The second order scheme designed and implemented uses a LaGrange Interpolating polynomial to extrapolate the future velocity trend as a function of time which is then integrated over the time step period for all 3 spatial dimensions to estimate the position at the text time step. The general form of the LaGrange interpolating polynomial is given is equations (1) and (2)

Where

Given 3 know previous know velocities v1, v2, v3 at t1, t2 and t3 respectively, the velocity-time curve interpolating polynomial is given by equation (3)

The time-step between recalculations of the Biot-Savart law is fixed during the simulation, so the three know previous velocities occur at equally spaced time steps, given a fixed time step of tts, the values of t for which velocities are known are given in equation (4) where t is is the current time.

For each time step a new interpolating polynomial is calculated, and the function integrated over the range t to t+tts, to simplify the calculation the polynomial is instead centred around 0, for this case the known velocities reduces to equation (5), this polynomial is integrated over the range 0 to tts.

Centring the polynomial on 0 reduces the complexity of equation 3 by removing the term x1, equation (5) shows equation (4) with substituted values for t1, t2 and t3

Equation (5) further simplifies to equation (6)

This can be expressed as a second degree polynomial of the form of equation (7)

Where the coefficients a, b and c are given by equations 8, 9 and 10 respectively

The change in position of all the blobs is calculated by the integration of this function over the range 0 to tts, this is shown in equation 11

Expanding equation (11) by replacing the coefficients of the polynomial a, b and c with substitutions from equations (7) through to (10) results in equation (12).

This scheme is applied for all three spatial dimensions, this is shown in matrix form in equation (13)

Given increasingly small time steps, the interpolating polynomial may tend to produce

# Temporal Discretization – Comparison of Schemes

Both schemes require a constant

